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PREPARATION OF AN EXERCISE ON HISTORICAL METHODS IN ARITHMETIC

HERMAN T. LUKENS

This material was collected without any purpose of using it as a morning exercise. The Russian-peasant method of multiplying without the use of the multiplication table was first tried with the children of the fifth grade and on several adults. With Miss Hall's hearty interest to welcome it, the fifth grade visited the fourth grade and spent a period in demonstrating it to them. The interest the children showed was sufficient to encourage me to present the lattice method to the fifth grade. Each of these methods was used comparatively, the same multiplication being performed, first by the usual method, then by the Russian-peasant method, and thirdly by the lattice method. Each method tested the other methods. No pupil ventured to hand in work that did not prove. We improved in accuracy rapidly.

The children liked the work and asked for other methods of division. I showed them the scratch method or galley method. This forms an intermediate method between the short division and the long division. I believe it helped very materially in making plain the essential thought-work of division, which is the same in short division, scratch division and long division.

By this time the children began to expect that I could give them still other methods that would be as interesting and they asked for more. I showed them the method of multiplying by complements. I showed them a few magic squares, how to tell whether a number is divisible by 2, 3, 5, 9, or 10, without actually dividing it, and lastly I showed them the method of proof of all four operations by casting out the 9's.

Out of this interest and much willing practice, the children secured a large amount of familiarity with the number combinations and considerable skill in calculation.

It was at this stage that I proposed to them to make a morning exercise of it. They had already been telling their parents about the Russian-peasant method at home. I found some of them paying better attention in class when they realized for the first time that they could not go home and get father or mother to show them "how." It was necessary in preparation for the morning exercise to impress on them the need of speaking clearly, slowly, and of pointing to figures as they referred to them. The whole class was prepared, and any of them could have done any of the exercises. The day chosen for the exercise was Friday, January 3. The week before the holiday vacation, I gave a test to the whole class. All but about six had everything correct. I did not choose the best pupils to take part in the exercise. I chose those who, as I thought, needed it. They were definitely told the second school day before the exercise was given.

The day before the exercise and the morning of the exercise, I took the children to the assembly hall to get used to speaking in the large room. While the janitors were placing the chairs I had the weak whisperers try to speak above the noise and confusion, as Demosthenes did at the seashore. While they could not be heard half way across the room, it did them much good to test their voices under such conditions. They were in excellent good humor and mettle for the exercise.

The following is a report of the morning exercise as it was given.

COMPARATIVE METHODS IN THE FUNDAMENTAL OPERATIONS

Dr. Lukens. We are going to show you other ways of multiplying and dividing than the usual ways that are taught. It is not at all to be understood that these are offered as better ways, but they are interesting; they help us by comparison to understand better the usual ways, and they serve as a proof or test of the work by doing it in more than one way.

Theresa. First we are going to show you how the Russian peasant multiplies without using the multiplication table.

45 × 27
~~22 54~~
11 108
5 216
~~2 432~~
1 864
1,215

27 × 45
13 90
~~6 180~~
3 360
1 720
1,215

The first column is obtained by taking successively half of the number above. The second column is obtained by doubling each number successively to correspond with the quantities in the first column. You then cross out every line that corresponds to an *even* number in the first column. You add the rest of the second column and thus obtain the product of the two numbers.

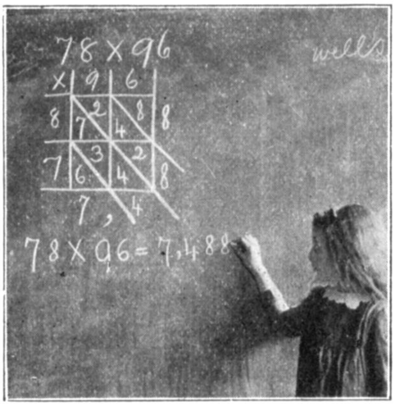
The first column is the *control*-column and has nothing else to do with the product. You do not add it. You take half of each number successively, rejecting any fraction, and writing only the whole numbers. Whenever the control column has an *even* number, cross out the entire line. Add the remaining numbers in the second column. Their sum is the product of the original numbers.

Frederick and Jean then did another example, one by the Russian-peasant method and the other by the usual method, for comparison.

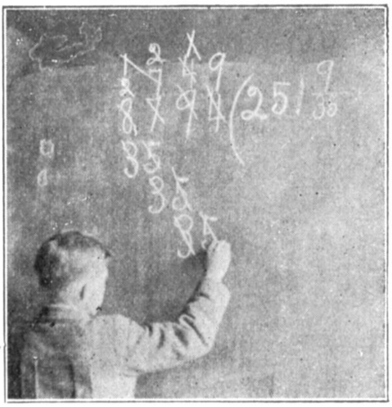
Janet. I am going to multiply 78 by 96, using the lattice method of multiplication.

The distinctive feature of the method is that you do not have to carry as you multiply, but only as you add up the diagonal columns.

Philipp and Elisabeth then did another example by both methods, for comparison.



Lattice Multiplication



Scratch Division

As the time for the exercise closed we were asked to continue on a following day and give other methods. This was done as follows:

Dr. Lukens. The other day an explanation of the reason for the Russian-peasant method was asked for. The reason may be worked out by algebra, but not in this morning exercise. I can suggest it however. If you let *n* represent the second number, you can see how the control numbers determine which numbers in the second column to add. Thus:

~~2 × n~~
1 2n
2n

3 × n
1 2n
3n

~~4 × n~~
~~2 2n~~
1 4n
4n

5 × n
~~2 2n~~
1 4n
5n

~~6 × n~~
3 2n
1 4n
6n

7 × n
3 2n
1 4n
7n

Halloween. I am going to use the scratch method of division in dividing 8,794 by 35.

The main point is to keep the columns correct. The divisor is written below the dividend, and the remainders above the dividend. Both are scratched as fast as they are used. Thus, 2 times 3 are 6, and 6 from 8 leaves 2, written above the 8; 2 times 5 are 10, and 10 from 27 leaves 17, written above the 27. The divisor is then moved over one place toward the right. Then, 35 is contained in 179 five times, and we say 5 times 3 are 15, and 15 from 17 leaves 2, written above the 7; 5 times 5 are 25, and 25 from 29 leaves 4, written above the 9. The divisor is rewritten one place to the right. Then 35 is contained in 44 once. We say 3 from 4 leaves 1, written above the 4, 5 from 14 leaves a remainder of 9. The numbers are canceled or scratched as soon as used, so as to keep clear what number one is working with at any moment and to prevent confusion.

William, Emily and Cony now did an example, one by short division, one by scratch division, and one by long division, to show that the operation in the mind is exactly the same in all division, but that in the short division only the quotient is written; in scratch division the remainders and the quotient; and in long division all is written, products, remainders and quotients.

Janet then multiplied by complements.

Lois then showed a magic square of 15. John proved an addition by casting out the 9's. Henry proved a subtraction and Mary a multiplication. Allan was in the midst of the proof of division by casting out the 9's when the closing bell rang.[1]

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[1] It has often seemed to me that pupils have no chance to *think,* when they are limited to one set form of work, as is usually the case in the four fundamental operations. They need to try other ways for the purpose of comparison. The smallest number of ideas we can think with is *two,* not one. Comparison is essential for thought. Our recently emasculated, simplified, and mechanized forms of efficiency in mathematics are at the expense of thought and understanding.